Answers are found at the bottom of the worksheet on a seperate page.

Question 1: Find the flaw in the following proof.

The sum of two odd integers is even.

Proof. Let x and y be odd integers. Since x is odd, there is an integer n such that $x = 2n + 1$. Since y is odd, there is an integer $y = 2n + 1$. So

$$x + y = 2n + 1 + 2n + 1 = 4n + 2 = 2(2n + 1).$$

Since n is an integer, so is $2n + 1$. So by definition, $x + y$ is even. □

Question 2: Show that Even + Odd = Odd

Question 3: Show that if a is odd, then a^2 is odd.

Question 4: Find the flaw in the following proof.

Every integer (Here, integer means non-decimal number) is odd.

Proof. Let x be an integer. Note that

$$x = 2\left(\frac{x}{2}\right) = 2\left(\frac{x}{2}\right) - 1 + 1 = 2\left(\frac{x}{2} - \frac{1}{2}\right) + 1.$$

Let $k = \frac{x}{2} - \frac{1}{2}$. So then $x = 2k + 1$. □

Question 5: What are the factors of 1001? What are the prime factors of 1001? Remember, 1 isn't prime. You're allowed to use a calculator.

Question 6: Show that $x^2 - 4$ is Composite when $x \geq 4$.

Remark: We introduce the division algorithm. Given a, b which are non-decimal numbers, we can write $a = qb + r$ where q, r are non-decimal numbers and $0 \leq r < b$.

For example, if $a = 14$ and $b = 5$, then $q = 2$ and $r = 4$. Then, $14 = 2(5) + 4$.

Why this is called an algorithm, I don't know. And while we will talk about this later, here's something cool.

a is composite is equivalent to saying that we can use the division algorithm to rewrite $a = qb$ with $r = 0$ for some $2 \leq b < a$.

Why is this important? IDK. But it's pretty cool! Now, if you want, consider this: How can we use the division algorithm to say when a number is prime?

Answer 1: We let $x = 2n + 1$ and $y = 2n + 1$. But this means that $x = y$. See this issue? We need to use seperate variables to show that both are different odd numbers. So let $x = 2n + 1$ and $y = 2m + 1$.

Remember, n and m are some number so that we can write x and y as odd numbers. This small caveat I forgot to mention in my video.

Note: Interestingly, the claim, or what we are trying to prove is correct. Keep in mind that something can be true but a proof can be false, so this is why verifying our proofs are important.

Answer 2:

Proof. Let $x = 2k$ be even and $y = 2l$ be odd. Then,

$x + y = 2k + 2l + 1 = 2(k + l) + 1$. The final number is odd. □

Answer 3:

Proof. Let $a = 2k + 1$ be odd. Then:

$a^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. The final number is odd. □

Answer 4: $\frac{x}{2}$ is not a non-decimal number, or an integer.

To see this, try $x = 2$. Then, $k = \frac{1}{2}$. But this is not a non-decimal number, or an integer, so the proof is false.

Answer 5: The factors of 1001 are: $1, 7, 11, 13, 77, 91, 143, 1001$.

The prime factors are: $7, 11, 13$.

Answer 6:

Proof. Use the following algebraic identity:

$$(x + y)(x - y) = x^2 - y^2$$

Then, when $y = 4$, we have that:

$$x^2 - 2^2 = (x + 2)(x - 2)$$

Then, we see that $1 < (x - 2) \leq (x + 2) < x^2$, so by definition, $x^2 - 4$ is composite. □